Unit 10 Independent Summer Packet

Name____

For each skill in this packet, there are examples, explanations and definitions to read followed by practice problems for you to complete.

Complex Fractions and Unit Rates

Fractions like – are called complex fractions. **Complex fractions** are fractions with a numerator, denominator, or both that are also fractions or decimals.

Example:

1. Simplify – . A fraction can also be written as a division problem.

– = 2 ÷ – Write the complex fraction as a division problem.

= – × – Multiply by the reciprocal of – which is – .

= – or 2 – Simplify.

So, – is equal to 2 – .

2 **Simplify.**

$\frac{\frac{8}{9}}{6} = \frac{8}{9} \div 6$ Write the complex fraction as a division problem.

$= \frac{8}{9} \times \frac{1}{6}$ Multiply by the reciprocal of 6.

$= \frac{8}{54}$ or $\frac{4}{27}$ Simplify.

2 **Mary is making pillows for her Life Skills class. She bought $2\frac{1}{2}$ yards of fabric. Her total cost was $15. W hat was the cost per yard?**

$\frac{15}{2\frac{1}{2}} = 15 \div 2\frac{1}{2}$ Write the complex fraction as a division problem.

$= 15 \div \frac{5}{2}$ Write the mixed number as an improper fraction.

$= 15 \times \frac{2}{5}$ Multiply by the reciprocal of $\frac{5}{2}$.

$= \frac{30}{5}$ or 6 Simplify. Mary spent $6 per yard of fabric.

PRACTICE: Simplify.

1. —

2. —

3. —

4. —

5. —

6. —

7. —

8. —

Proportional and Nonproportional Relationships

> Two related quantities are **proportional** if they have a constant ratio between them. If two related quantities do not have a constant ratio, then they are **nonproportional.**

Example 1

The cost of one CD at a record store is $12. Create a table to show the total cost for different numbers of CDs. Is the total cost proportional to the number of CDs purchased?

Number of CDs	1	2	3	4
Total Cost	$12	$24	$36	$48

———— = — = — = — = — = $12 per CD

Divide the total cost for each by the number of CDs to find a ratio. Compare the ratios.

Since the ratios are the same, the total cost is proportional to the number of CDs purchased.

Example 2

The cost to rent a lane at a bowling alley is $9 per hour plus $4 for shoe rental. Create a table to show the total cost for each hour a bowling lane is rented if one person rents shoes. Is the total cost proportional to the number of hours rented?

Number of Hours	1	2	3	4
Total Cost	$13	$22	$31	$40

———— ? — or 13 — or 11 — or 10.34 — or 10

Divide each cost by the number of hours.

Since the ratios are not the same, the total cost is nonproportional to the number of hours rented with shoes.

PRACTICE:

1. **PICTURES** A photo developer charges $0.25 per photo developed. Is the total cost proportional to the number of photos developed?

2. **SOCCER** A soccer club has 15 players for every team, with the exception of two teams that have 16 players each. Is the number of players proportional to the number of teams?

Write the ratios in the table to show the relationship between each set of values.

1.

Number of Hours	1	2	3	4
Total Amount Earned	$15	$30	$45	$60
Ratios				

2.

Number of Packages	1	2	3	4
Total Cost	$11	$20	$29	$38
Ratios				

3.

Number of Classrooms	1	2	3	4
Total Students	24	48	72	92
Ratios				

For Exercises 4–8 use the table of values. W rite *proportional* or *nonproportional.*

4.

Number of Hours	1	2	3	4
Total Amount Earned	$0.99	$1.98	$2.97	$3.96

5.

Number of Hours	1	2	3	4
Total Amount Earned	$17.25	$35.50	$50.75	$70

6.

Number of Hours	1	2	3	4
Number of Pages Read in Book	37	73	109	145

7.

Number of Lunches	1	2	3	4
Total Cost	$2.75	$5.50	$8.25	$11

8. Fred is ordering pies for a family reunion. Each pie costs $4.50. For orders smaller than a dozen pies, there is a $5 delivery charge. Is the cost proportional to the number of pies ordered? Use a table of values to explain your reasoning.

Graph Proportional Relationships

A way to determine whether two quantities are proportional is to graph them on a coordinate plane. If the graph is a straight line through the origin, then the two quantities are proportional.

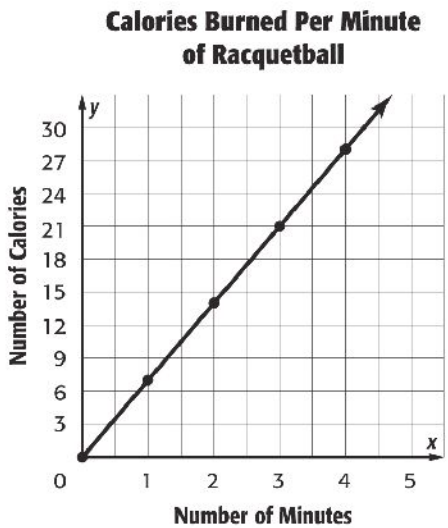
Example 1

A racquetball player burns 7 Calories a minute. Determine whether the number of Calories burned is proportional to the number of minutes played by graphing on the coordinate plane.

Step 1 Make a table to find the number of Calories burned for 0, 1, 2, 3, and 4 minutes of playing racquetball.

Time (min)	0	1	2	3	4
Calories Burned	0	7	14	21	28

Step 2 Graph the ordered pairs on the coordinate plane. Then connect the ordered pairs.



The line passes through the origin and is a straight line. So, the number of Calories burned is proportional to the number of minutes of racquetball played.

Ratios and Proportional Reasoning

Graph Proportional Relationships
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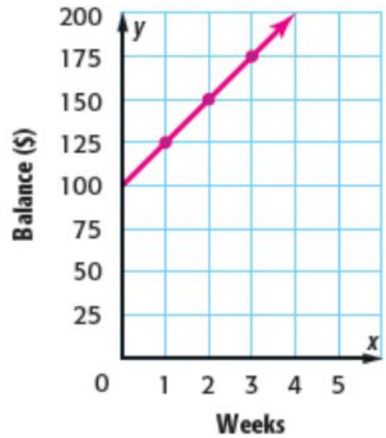
Model with M athematics **Determine whether the relationship between**
2

the two quantities shown in the table are proportional by graphing on the coordinate plane. Explain your reasoning.

Savings Account	
Week	Account Balance ($)
1	125
2	150
3	175

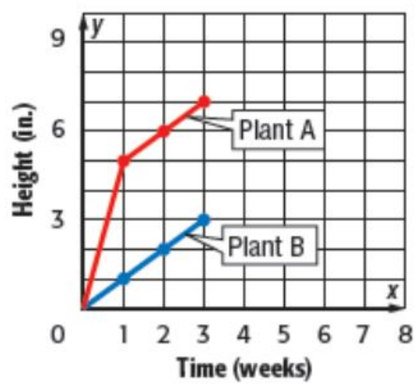


The relationship between the two quantities shown in the table are not proportional. They are not proportional because when they are graphed as ordered pairs on the coordinate plane, the line does not pass through the origin.

3
2 **The height of two plants is recorded after 1, 2, and 3 weeks as shown in the graph at the right. W hich plants' growth represents a proportional relationship between time and height? Explain.**

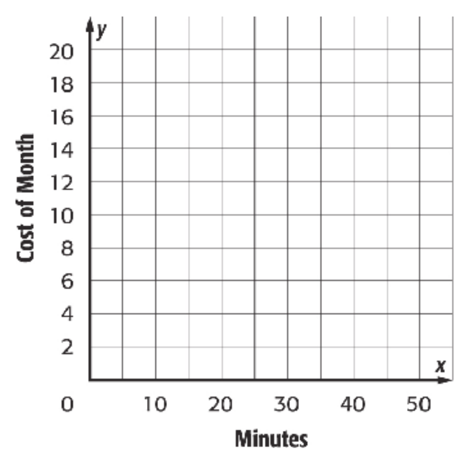
The growth rate of Plant A is not proportional because the line is not straight.

The growth rate of Plant B is proportional because when the data is graphed on the coordinate plane, the line formed is a straight line that passes through the origin.

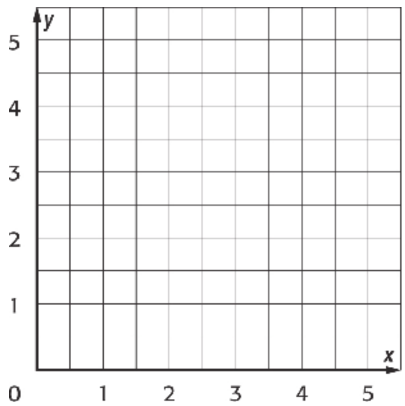


PRACTICE:

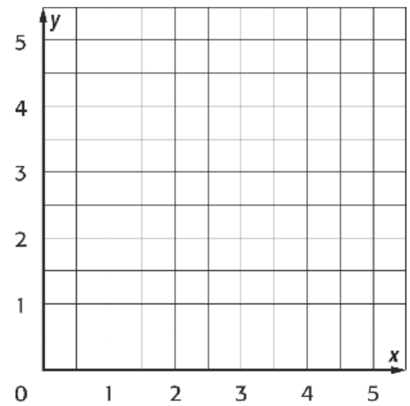
1. Shontell spends $7 a month plus $0.10 per minute. Determine whether the cost per month is proportional to the number of minutes by graphing on the coordinate plane.



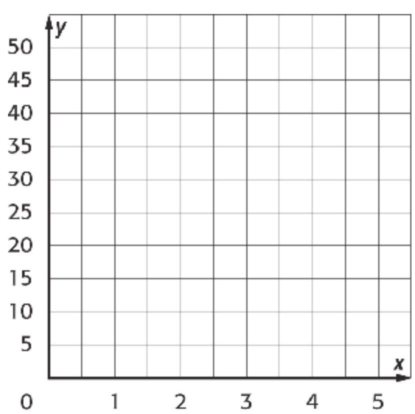
1. BAKING Rachel baked 3 cakes in 2 hours, 4 cakes in 3 hours, and 5 cakes in 4 hours. Determine whether the number of cakes baked is proportional to the number of hours.



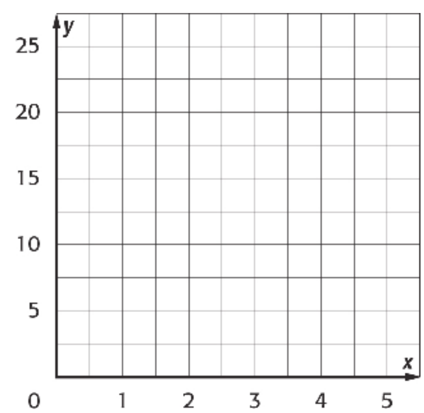
2. RAINFALL It rained 2 inches in one hour, then after two hours, it had rained a total of 3 inches. After four hours, it had rained a total of 5 inches. Determine whether the number of inches of rainfall is proportional to the number of hours.



3. CALORIES A person can burn 8 Calories per minute of running. Determine whether the number of Calories is proportional to the number of minutes.



4. PROFIT If Stephanie sells 3 necklaces, she earns a profit of $5. If she sells 4 necklaces, her profit is $10. Five necklaces sold gives her a profit of $15 and six necklaces sold gives her a profit of $20. Determine whether the amount of profit is proportional to the number of necklaces sold.



Solving Proportions

A **proportion** is an equation that states that two ratios are equivalent. To determine whether a pair of ratios forms a proportion, use cross products. You can also use cross products to solve proportions.

Example 1

Determine whether the pair of ratios — and — form a proportion.

Find the cross products.

$\frac{20}{24} \stackrel{?}{=} \frac{12}{18}$ → 24 • 12 = 288

→ 20 • 18 = 360

Since the cross products are not equal, the ratios do not form a proportion.

Example 2

Solve — = —.

— = —	**1. Write the equation.**
	***2.* Find the cross products.**
— = —	**3. Divide each side by the coefficient, 30.**
28 = *k*	**The solution is 28.**

PRACTICE: Determine whether each pair of ratios forms a proportion.

1. — , — **2.** – , — **3.** — , —

Solve each proportion.

4. – = — **5.** — = — **6.** – = — **7.** — = — **8.** – = —

Set up a proportion to solve for the missing value.

5. . TYPING Ingrid types 3 pages in the same amount of time that Tanya types 4.5 pages. If Ingrid and Tanya start typing at the same time, how many pages will Tanya have typed when Ingrid has typed 11 pages?

6. AMUSEMENT PARKS The waiting time to ride a roller coaster is 20 minutes when 150 people are in line. How long is the waiting time when 240 people are in line?

Constant Rate of Change

1
2 **Find the constant rate of change for the table.**

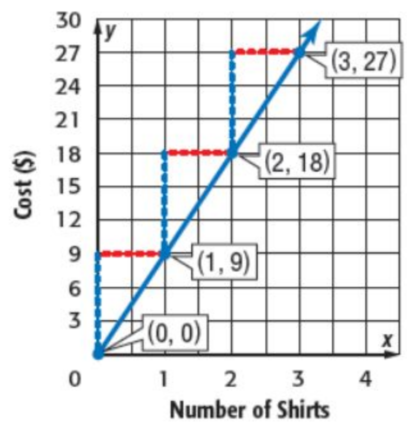
Time (s)	Distance (m)
1	6
2	12
3	18
4	24

Find the unit rate to find the constant rate of change.

$$\frac{\text{change in meters}}{\text{change in seconds}} = \frac{6\text{m}}{1\text{s}}$$

So, the constant rate of change is 6 meters per second.

3
2 **The graph shows the cost of purchasing T-shirts. Find the constant rate of change for the graph. Then explain what points (0, 0) and (1, 9) represent.**



Find the constant rate of change from the graph.

$$\frac{\text{change in cost}}{\text{change in number of T-shirts}} = \frac{9-0}{1-0} = \frac{9}{1}$$

So, the constant rate of change is $9 per T-shirt;

The point (0, 0) represents 0 T -shirts purchased and 0 dollars spent. The point (1, 9) represents 9 dollars spent for 1 T-shirt.

A **rate of change** is a rate that describes how one quantity changes in relation to another.

A **constant rate of change** is the rate of change of a linear relationship.

Example 1
Find the constant rate of change for the table.

Students	Number of Textbooks
5	15
10	30
15	45
20	60

The change in the number of textbooks is 15. The change in the number of students is 5

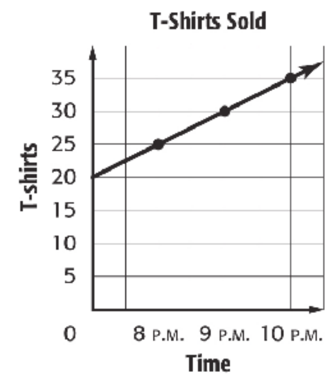
______________ = ________ The number of textbooks increased by 15 for every 5 students.

= ________ Write as a unit rate.

So, the number of textbooks increases by 3 textbooks per student.

Example 2

The graph represents the number of T-shirts sold at a band concert. Use the graph to find the constant rate of change in number per hour.



To find the rate of change, pick any two points on the line, such as (8, 25) and (10, 35).

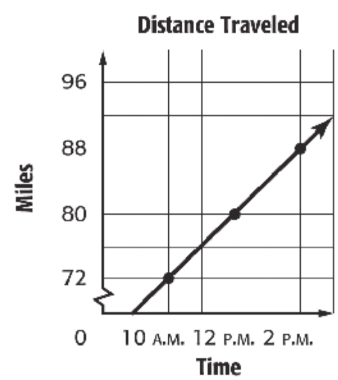
__________ = ( ) / ( ) = — or 5 T -shirts per hour

PRACTICE: Find the each constant rate of change.

1.

Side Length	Perimeter
1	4
2	8
3	12
4	16

2.



3.

Number of Pounds of Ham	Cost ($)
0	0
3	12
6	24
9	36

5.

Number of Hours W orked	Money Earned ($)
4	80
6	120
8	160
10	200

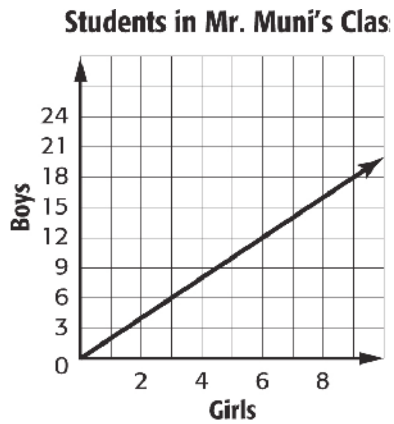
4.

Days	Plant Height (in.)
7	4
14	11
21	18
28	25

6.

Months	Money Spent on Cable TV
2	82
4	164
6	246
8	328

7.



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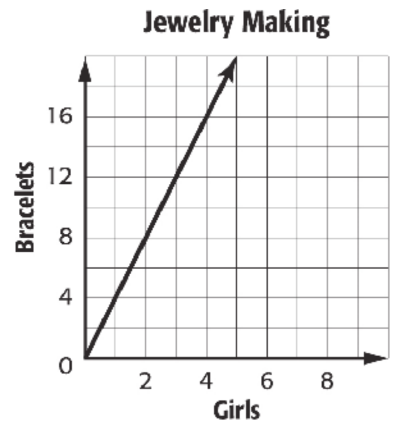


9. **SEAGULLS** At 1 P.M., there were 16 seagulls on the beach. At 3 p.m., there were 40 seagulls. What is the constant rate of change?

Slope

Slope is the rate of change between any two points on a line.

slope = ________ or ____________ or ___

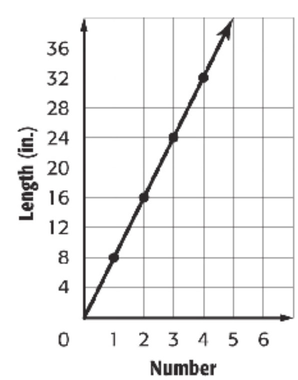
Example

The table shows the length of a patio as blocks are added.

Number of Patio Blocks	0	1	2	3	4
Length (in.)	0	8	16	24	32

Graph the data. Then find the slope of the line.

Explain what the slope represents.



slope = $\frac{\text{change in}}{\text{change in}}$ Definition of slope

= _____ Use (1, 8) and (3, 24).

= __ _____

= – Simplify.

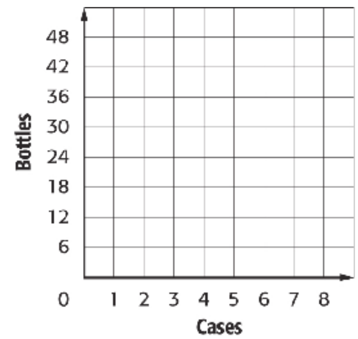
So, for every 8 inches, there is 1 patio block.

PRACTICE:

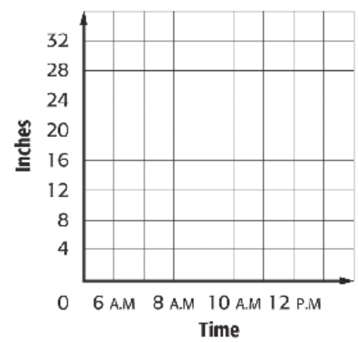
Graph the data. Then find the slope of the line. Explain what the slope represents.

1. The table shows the number of juice bottles per case.

Cases	1	2	3	4
Juice Bottles	12	24	36	48



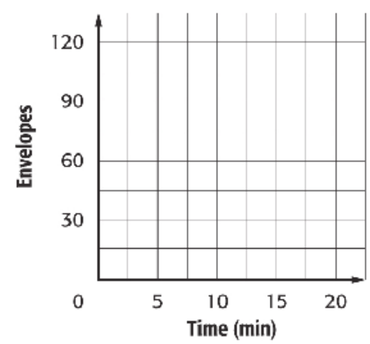
2. At 6 A.M., the retention pond had 28 inches of water in it. The water receded so that at 10 A.M. there were 16 inches of water left.



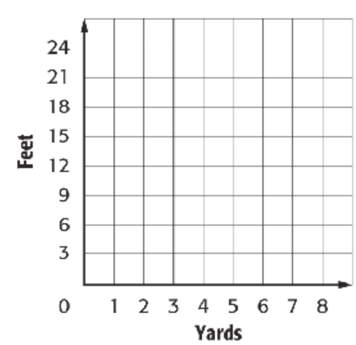
Graph the data. Then find the slope. Explain what the slope represents.

3. **ENVELOPES** The table shows the number of envelopes stuffed for various times.

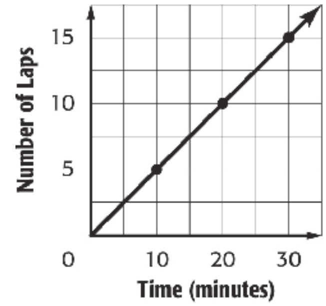
Time (min)	5	10	15	20
Envelopes Stuffed	30	60	90	120



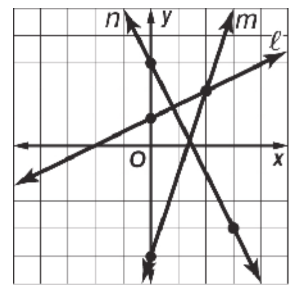
4. **MEASUREMENT** There are 3 feet for every yard.



4. Use the graph that shows the number of laps completed over time. Find the slope of the line.

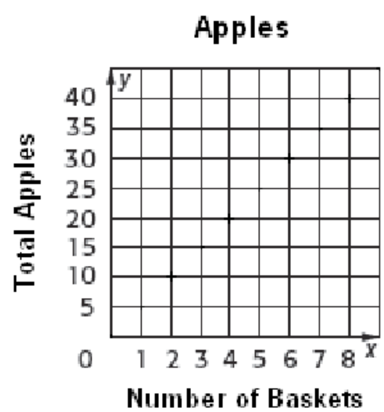


5. Which line is the steepest? Explain using the slopes of lines l, m, and n.



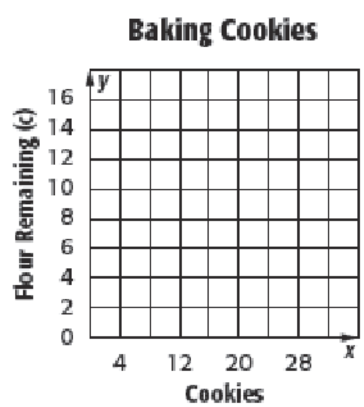
6. The table below shows the number of apples per basket at a farmer's market. Graph the data. Then find the slope of the line. Explain what the slope represents.

Baskets	2	4	6	8
Apples	10	20	30	40



7. The table below shows the amount of flour left after baking certain amounts of cookies. Graph the data. Then find the slope of the line. Explain what the slope represents.

Number of Cookies	0	8	16	24	32
Flour Remaining (c)	16	12	8	4	0



Direct Variation(Direct Proportion)

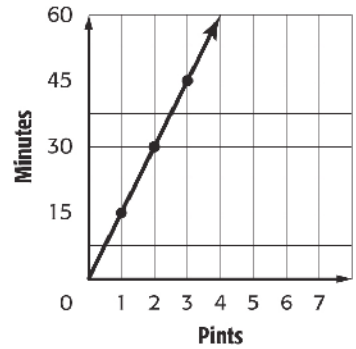
When two variable quantities have a constant ratio, their relationship is called a **direct variation**.

The constant ratio is called the **constant of proportionality**.

The equation for a direct proportion is always in y=kx form where k is the constant of the proportionality (slope or rate of change).

Example 1

The time it takes Lucia to pick pints of blackberries is shown in the graph. Determine the constant of proportionality.



Since the graph forms a line, the rate of change is constant. Use the graph to find the constant of proportionality.

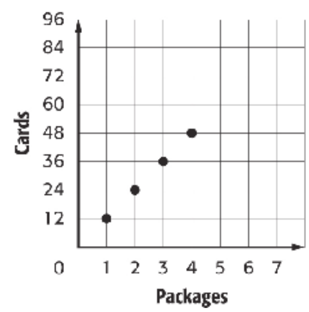
$\frac{\qquad}{\text{number of pints}}$ = —   — or —   — or —

It takes 15 minutes for Lucia to pick 1 pint of blackberries.

Example 2

There are 12 trading cards in a package. M ake a table and graph to show the number of cards in 1, 2, 3, and 4 packages. Is there a constant rate? a direct variation?

Numbers of Packages	1	2	3	4
Number of Cards	12	24	36	48



Because there is a constant increase of 12 cards, there is a constant rate of change. The equation relating the variables is $y = 12x$, where y is the number of cards and x is the number of packages. This is a direct variation. The constant of proportionality is 12.

Exercises

1. **SOAP** Wilhema bought 6 bars of soap for $12. The next day, Sophia bought 10 bars of the same kind of soap for $20. What is the cost of 1 bar of soap?

2. **COOKING** Franklin is cooking a 3-pound turkey breast for 6 people. If the number of pounds of turkey varies directly with the number of people, make a table to show the number of pounds of turkey for 2, 4, and 8 people.

2 **Veronica is mulching her front yard. The total weight of mulch varies directly with the number of bags of mulch. W hat is the rate of change?**

Since the graph forms a line, the rate of change is constant. Use the graph to find the constant of proportionality.

$$\frac{\text{weight (lb)}}{\text{bags}} = \frac{30}{1}$$

The rate of change is 30 pounds per bag.

2 **Determine whether the linear function is a direct variation. If so, state the constant of proportionality.**

Minutes, *x*	185	235	275	325
Cost, *y*	60	115	140	180

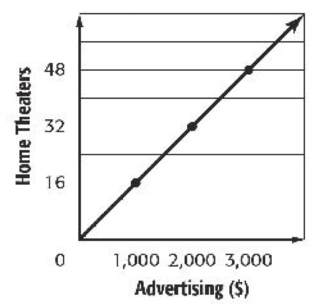
Compare the ratios to check for a common ratio.

$$\frac{\text{cost}}{\text{minutes}} \quad \frac{60}{185} \text{ or } \frac{12}{37} \neq \frac{115}{235} \text{ or } \frac{23}{47} \neq \frac{140}{275} \text{ or } \frac{28}{55} \neq \frac{180}{325} \text{ or } \frac{36}{65}$$

The linear function shown in the table is not a direct variation because there is no common ratio.

PRACTICE:

1. HOME THEATER The number of home theaters a company sells varies directly as the money spent on advertising. How many home theaters does the company sell for each $500 spent on advertising?



2. DUNE BUGGY Beach Travel rents dune buggies for $50 for 4 hours or $75 for 6 hours. What is the hourly rate?

3. FERTILIZER Leroy uses 20 pounds of fertilizer to cover 4,000 square feet of his lawn and 50 pounds to cover 10,000 square feet. How much does he need to cover his entire yard which has an area of 26,400 square feet?

Determine whether each linear function is a direct variation. If so, state the constant of variation. Hint: Since y=kx, k=y/x

4.

Gallons, *x*	6	8	10	12
Miles, *y*	180	240	300	360

5.

Time (min), *x*	10	11	12	13
Temperature, *y*	82	83	84	85

6.

Number of Payments, *x*	6	11	16	21
Amount Paid, *y*	$1,500	$2,750	$4,000	$5,250

If *y* varies directly with *x*, write an equation for the direct variation. Then find each value.

7. If $y = -4$ when $x = 10$, find y when $x = 5$.

8. If $y = 12$ when $x = -15$, find y when $x = 2$.

9. Find x when $y = 18$, if $y = 9$ when $x = 8$.

The Percent Proportion

A **percent proportion** compares part of a quantity to a whole quantity for one ratio and lists the percent as a number over 100 for the other ratio.

part ?
whole ? – = —— percent

Example 1

What percent of 24 is 18?

– = —— Percent proportion

Let $n\%$ represent the percent.

— = —— Write the proportion.

$18 \times 100 = 24 \times n$ Find the cross products.

$1{,}800 = 24n$ Simplify.

—— = —— Divide each side by 24.

$75 = n$

So, 18 is 75% of 24.

Example 2

What number is 60% of 150?

– = —— Percent proportion

Let $n\%$ represent the percent.

—— = —— Write the proportion.

$n \times 100 = 150 \times 60$ Find the cross products.

$100n = 9{,}000$ Simplify.

—— = —— Divide each side by 100.

$n = 90$

So, 90 is 60% of 150.

PRACTICE: Find each number. Round to the nearest tenth if necessary.

1. What number is 25% of 20? **2.** What percent of 50 is 30? **3.** 30 is 75% of what number?

4. 40% of what number is 36?

5. What number is 20% of 625?

6. 12 is what percent of 30?

7. **ALLOWANCE** Mallorie has $3 in her wallet. If this is 10% of her monthly allowance, what is her monthly allowance?

8. **WEDDING** Of the 125 guests invited to a wedding, 104 attended the wedding. What percent of the invited guests attended the wedding?

9. **CAMERA** The memory card on Melcher's digital camera can hold about 430 pictures. Melcher used 18% of the memory card. About how many pictures did Melcher take? Round to the nearest whole number.

Sales Tax, Tips, and Markup

> **Sales Tax** is a percent of the purchase price and is an amount paid in addition to the purchase price.
>
> **Tip,** or **gratuity,** is a small amount of money in return for service. The amount a store increases the price of an item by is called the **markup**.

Example 1

SOCCER Find the total cost of a $17.75 soccer ball if the sales tax is 6%.

Method 1

First, find the sales tax.

6% of $17.75 = 0.06 • 17.75

˜ 1.07

The sales tax is $1.07.

Next, add the sales tax to the regular price.

1.07 + 17.75 = 18.82

The total cost of the soccer ball is $18.82.

Method 2

100% + 6% = 106% Add the percent of tax to 100%.

The total cost is 106% of the regular price.

106% of $17.75 = 1.06 • 17.75

˜ 18.82

Example 2

MEAL A customer wants to leave a 15% tip on a bill for $18.50 at a restaurant.

Method 1 Add tip to regular price.

First, find the tip.

15% of $18.50 = 0.15 • 18.50

= 2.78

Next, add the tip to the bill total.

$18.50 + $2.78 = $21.28

Method 2 Add the percent of tip to 100%.

100% + 15% = 115% Add the percent of tip to 100%.

The total cost is 115% of the bill.

115% of $18.50 = 1.15 • 18.50

= 21.28

The total cost of the bill is $21.28.

PRACTICE: **Find the total cost to the nearest cent.**

1. $22.95 shirt, 6% tax

2. $24 lunch, 15% tip

3. $10.85 book, 4% tax

4. $97.55 business breakfast, 18% tip

5. $59.99 DVD box set, 6.5% tax

6. $37.65 dinner, 15% tip

Financial Literacy

Simple interest is the amount of money paid or earned for the use of money. To find simple interest *I*, use the formula $I = prt$. Principal *p* is the amount of money deposited or invested. Rate *r* is the annual interest rate written as a decimal. Time *t* is the amount of time the money is invested in years.

Example 1

Find the simple interest earned in a savings account where $136 is deposited for 2 years if the interest rate is 7.5% per year.

$I = prt$	Formula for simple interest
$I = 136 \cdot 0.075 \cdot 2$	Replace *p* with $136, *r* with 0.075, and *t* with 2.
$I = 20.40$	Simplify.

The simple interest earned is $20.40.

Example 2

Find the simple interest for $600 invested at 8.5% for 6 months.

6 months = — or 0.5 year	Write the time in years.
$I = prt$	Formula for simple interest
$I = 600 \cdot 0.085 \cdot 0.5$	*p* = $600, *r* = 0.085, *t* = 0.5
$I = 25.50$	Simplify.

The simple interest is $25.50.

PRACTICE: Find the simple interest earned to the nearest cent for each principal, interest rate, and time.

1. $300, 5%, 2 years

2. $650, 8%, 3 years

3. $575, 4.5%, 4 years

4. $735, 7%, 2 – years

5. $1,665, 6.75%, 3 years

6. $2,105, 11%, 1 – years

7. \$903, 8.75%, 18 months

8. \$4,275, 19%, 3 months

Scale Drawings

Find the actual distance between Columbia and Charleston.

Use a ruler to Measure.

The distance between the two cities on the map is 3.8 centimeters. Write and solve a proportion using the scale. Let *d* represent the actual distance between the cities.

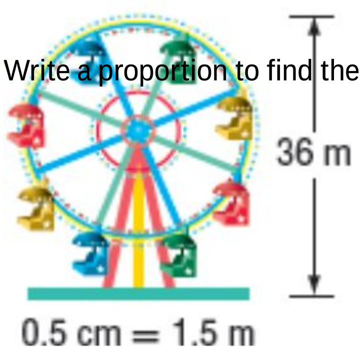
$$\frac{\text{map} \rightarrow}{\text{actual} \rightarrow} \frac{1 \text{ cm}}{27 \text{ mi}} = \frac{3.8 \text{ cm}}{d \text{ mi}} \frac{\leftarrow \text{map}}{\leftarrow \text{actual}}$$

$$1 \times d = 3.8 \times 27$$

$$d = 102.6$$

So, the actual distance between Columbia and Charleston is 102.6 miles.

2 **Find the length of the model. Then find the scale factor.** Write a proportion to find the length.



$$\frac{0.5 \text{ cm}}{1.5 \text{ m}} = \frac{x \text{ cm}}{36 \text{ m}}$$

$$0.5 \bullet 36 = 1.5 \bullet x$$

$$18 = 1.5x$$

So,the model is 12 centimeters tall.

$$\frac{18}{1.5} = \frac{1.5x}{1.5}$$

$$12 = x$$

To find the scale factor, first convert centimeters to meters: 0.5 cm = 0.005 m. Next make a ratio and simplify.

$\frac{0.005}{1.5} = \frac{5}{1500}$ or $\frac{1}{300}$ *Hint: look at the units given in the key. 1m=100cm*

The scale factor is $\frac{1}{300}$. The scale factor is how many times larger or smaller the model is than the actual object.

PRACTICE:

ARCHITECTURE The scale on a set of architectural drawings for a house is – inch = – feet. Find the length of each part of the house.

	Room	Drawing Length	Actual Length
1.	Living Room	5 inches	
2.	Dining Room	4 inches	
3.	Kitchen	5 – inches	
4.	Laundry Room	3 – inches	
5.	Basement	10 inches	
6.	Garage	8 – inches	

ARCHITECTURE As part of a city building refurbishment project, architects have constructed a scale model of several city buildings to present to the city commission for approval. The scale of the model is 1 inch = 9 feet.

7. The courthouse is the tallest building in the city. If it is 7 – inches tall in the model, how tall is the actual building?

8. The city commission would like to install new flagpoles that are each 45 feet tall. How tall are the flagpoles in the model?

9. In the model, two of the flagpoles are 4 inches apart. How far apart will they be when they are installed?

10. The model includes a new park in the center of the city. If the dimensions of the park in the model are 9 inches by 17 inches, what are the actual dimensions of the park?

11. Find the scale factor of the model in question 10.